Thermal Management of Large Scale Optical Systems

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Airborne Sensing and Mapping
Operating Conditions of Airborne Cameras

- Flight height \( (0, \ldots, 3, \ldots, 9) \) km,
  - equivalent to
- Pressure \( (760, \ldots, 526, \ldots, 231) \) Torr
- Temperature \( (-30 \ldots +60) \) °C

- Narrow specifications of
  - Image registration : focal length, distortion
  - Image quality : resolution, MTF
Airborne Lens

- 420-900 nm spectral range
- Resolution ~ 150 lp/mm
- Wide angle
- Large depth of focus
- Registration accuracy 1 µm
- *Thermal & pressure stabilization*
Technical Solutions

- Control of performance degradations
  - Pressure variations
    - Mechanical sealing of the lens
  - Temperature variations
    - Two step process
      - Stationary: Optomechanical athermalization
      - Non stationary: Thermal management
Athermalization

Swiss watchmaker trick

Zerodur glass ceramics

\[ \frac{\Delta V}{\Delta T} = \alpha_1 V_1 + \alpha_2 V_2 = 0 \]

- \( \alpha_1 \) [Crystalline] < 0
- \( \alpha_2 \) [Amorphous] > 0

\[ \Rightarrow V_1 / V_2 = -\frac{\alpha_2}{\alpha_1} > 0 \]

\[ \frac{\Delta L}{\Delta T} = \alpha_1 L_1 - \alpha_2 L_2 = 0 \]

\[ \Rightarrow L_1 / L_2 = \frac{\alpha_2}{\alpha_1} \]


### Optomechanical Athermalization

#### Lens group

\[ \Delta L / \Delta T = \alpha_1 L_1 - \alpha_2 L_2 = c \neq 0 \]

- \( \alpha_1 \) [Aluminum]
- \( \alpha_2 \) [Invar]

\[ L_1 / L_2 = (c / L_2 + \alpha_2) / \alpha_1 \]

#### Optical system:

Several lens groups are mutually moved to fulfil:

- \( \Delta F / \Delta T = 0; \) Focal length
- \( \Delta s' / \Delta T = 0; \) Best focus
Image Quality for Different Image Zones

![Diagram showing image quality for different image zones with graphs illustrating the center and edge quality over time.](image-url)
Non stationary: Thermal Management

Questions

• How do thermal gradients influence the optical performance?

• When does the lens reach full performance, e.g. a lens at –20 °C is brought into a surrounding of +25 °C?

• How to thermally speed up to reach the fully operational state?
Mathematical Modeling

- Realistic modeling using
  - Each lens is modeled as a cylinder
  - But with the material parameter of the glass body $\kappa$
  - And the mechanical interface glass to mounting $d$

Differential Equation

$$\frac{\partial u}{\partial t} = \kappa \cdot \Delta u = \kappa \cdot \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

$u = \text{temperature}$
Material Parameters

\[ \kappa = \frac{\lambda}{c \rho} : \text{heat conductivity [m}^2\text{s}^{-1}] \]

\[ \lambda : \text{thermal conductivity [Wm}^{-1}\text{K}^{-1}] \]

\[ c : \text{specific heat [J kg}^{-1}\text{K}^{-1}] \]

\[ \rho : \text{density [kg m}^{-3}] \]

<table>
<thead>
<tr>
<th></th>
<th>N-LASF41</th>
<th>N-LF5</th>
</tr>
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<tbody>
<tr>
<td>\lambda</td>
<td>0.790</td>
<td>1.060</td>
</tr>
<tr>
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<td>810</td>
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</tr>
<tr>
<td>\kappa</td>
<td>3.3 \cdot 10^{-7}</td>
<td>5.1 \cdot 10^{-7}</td>
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</table>
Normalized Parameters

\[ \tilde{r} = \frac{r}{R}, \quad T = \frac{R^2}{\kappa}, \quad \tilde{t} = \frac{t}{T}, \quad \tilde{D} = \frac{D}{R}, \quad \tilde{u}(\tilde{r}, \phi, z; \tilde{t}) = \frac{u(r, \phi, z; t) - u_m}{u_m - u_0} \]

- **Characteristic time**
- **Characteristic thickness**
- **Normalized radius**
- **Normalized time**
- **Normalized temperature**

**Mechanical interface**

\[ D = d \cdot \frac{\lambda}{\lambda_{\text{glue}}} \]

\[ d = \text{glue thickness} \]
Simplifying Assumptions

- (i) The lens only exchanges heat with the mounting
- (ii) The temperature of the lens is constant in the direction of its optical axis (*no* $z$-*dependence*)
- (iii) The mounting is radially symmetric (*no* $\phi$-*dependence*)
- (iv) The mounting keeps its temperature independently of the quantity of heat it must deliver to the glass
  
  or
  
  (iv') The mounting holds the heat flux constant independently of the temperature of the glass
Heat Diffusion with constant Mounting Temperature

**Boundary conditions**

\[ u(r, 0) = u_0 \]

\[ D \cdot \frac{\partial u}{\partial r}(R, t) + u(R, t) = u_m \]

**Solution**

\[ \tilde{u}(\tilde{r}, \tilde{t}) = \sum_{j=1}^{\infty} \frac{-2 e^{-k_j^2 \tilde{t}}}{(\tilde{D}^2 k_j^2 + 1) \cdot k_j J_1(k_j)} J_0(k_j \tilde{r}) \]

\[ \tilde{D} \cdot k_j \cdot J_0'(k_j) + J_0(k_j) = 0 \]
## Characteristic Times

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<td>( \kappa = \lambda/(c \rho) )</td>
<td>3.3 ( \times ) ( 10^{-7} )</td>
<td>5.1 ( \times ) ( 10^{-7} ) ( \text{m}^2\text{s}^{-1} )</td>
</tr>
<tr>
<td>( R )</td>
<td>0.05</td>
<td>0.025 ( \text{m} )</td>
</tr>
<tr>
<td>( T = R^2 / \kappa )</td>
<td>2.1</td>
<td>0.34 ( \text{h} )</td>
</tr>
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Constant Heat Flux

Boundary conditions

\[ u(r, 0) = u_0 \]

\[ \lambda \cdot \frac{\partial u}{\partial r}(R, t) = q \]

\[ u = \text{temperature} \]

![Graph showing temperature as a function of radial distance for different times.]

Solution

\[ \tilde{u}(\tilde{r}, \tilde{t}) = \frac{\tilde{r}^2}{2} + 2\tilde{t} - \frac{1}{4} \sum_{j=1}^{\infty} \frac{2e^{-k_j^2\tilde{t}}}{k_j^2 J_0(k_j)} J_0(k_j \tilde{r}) \]

\[ U = \frac{qR}{\lambda}, \quad \tilde{u}(\tilde{r}, \tilde{t}) = \frac{u(r, t) - u_0}{U} \]
Turning the Heat Flux ‘on and off’

Boundary conditions

\[ u(r,0) = u_0 \]

\[ \lambda \cdot \frac{\partial u}{\partial r}(R,t) = \begin{cases} q, & 0 < t \leq t_0 \\ 0, & t > t_0 \end{cases} \]

Solution

\[ \tilde{u}(\tilde{r}, \tilde{t}) = \begin{cases} \frac{\tilde{r}^2}{2} + \frac{2\tilde{t}}{4} - \sum_{j=1}^{\infty} \frac{2e^{-k_j^2\tilde{t}_0}}{k_j J_0(k_j)} J_0(k_j \tilde{r}), & \tilde{t} < \tilde{t}_0 \\ 2\tilde{t}_0 + \sum_{j=1}^{\infty} \frac{2}{k_j^2 J_0(k_j)} \left(1 - e^{-k_j^2(\tilde{t} - \tilde{t}_0)}\right)e^{-k_j^2(\tilde{t} - \tilde{t}_0)} J_0(k_j \tilde{r}), & \tilde{t} > \tilde{t}_0 \end{cases} \]

\[ t_0 = T' \]
Heating a Part of the Cylinder Mantle

Boundary conditions

\[ u(r,0) = u_0 \]

\[
d \frac{\lambda}{h_m} \frac{h_L}{\lambda_{\text{glue}}} \frac{\partial u}{\partial r} (R,t) + u(R,t) - \left( u_m + H(t_0 - t) \cdot q \frac{h_h}{h_m \lambda_{\text{glue}}} \frac{d}{d} \right) = 0
\]

\[ \tilde{u}(\tilde{r}, \tilde{t}) = \frac{u(r,t) - u_m}{u_e - u_0} \]

\[ u_e := u_m + q \frac{h_h}{h_m \lambda_{\text{glue}}} \frac{d}{d} \]

Solution

\[
\tilde{u}(\tilde{r}, \tilde{t}) = \begin{cases} 
\gamma - \sum_{j=1}^{\infty} \frac{2 e^{-k_j^2 \tilde{t}}}{(D^2 k_j^2 + 1) \cdot k_j J_1(k_j)} J_0(k_j \tilde{r}), & \tilde{t} \leq \tilde{t}_0 \\
\sum_{j=1}^{\infty} \frac{2}{(D^2 k_j^2 + 1) \cdot k_j J_1(k_j)} \left[ \gamma e^{-k_j^2 (\tilde{t} - \tilde{t}_0)} - e^{-k_j^2 \tilde{t}} \right] \cdot J_0(k_j \tilde{r}), & \tilde{t} \geq \tilde{t}_0
\end{cases}
\]
Heating a Part of the Cylinder Mantle

\[
\tilde{D} = \frac{D}{R} = \frac{d}{R} \frac{\lambda}{\lambda_{\text{glue}}} \frac{h_t}{h_m}, \quad \tilde{u}(\tilde{r}, \tilde{t}) = \frac{u(r, t) - u_m}{u_e - u_0}, \quad \gamma = \frac{u_e - u_m}{u_e - u_0}
\]

\[
u = \frac{\nu_0}{T} = 0.1, \quad \gamma = 0.5, \quad \text{The wiggly line is for } t/T = 0.1001 \quad \text{and shows the Gibbs phenomenon.}
\]
Example: Disc 100 mm Diameter; N-LASF41

**Diffusion**

- **100 W / 30 min**
  - T: -20°C, T: 25°C, T: Char: 2°C, mpair: 2.12h

- **30 W / 15 min**

- **100 W / 45 min**
Hardware Concept: Individual Heating

Qi, ti
Summary

- Based on stationary athermalization
  - Measure temperature $T$ inside and outside of objective
  - Compute for each lens heat flux $Q_i$ and heat duration $t_i$
    - e.g. so that all lens centers are at the same temperature after the same time

- We presented, under simplifying assumptions, an analytically closed theory
  - Invariant to material and geometry parameters
  - Applicable to many different boundary conditions

- Start procedure for fine tuning of $Q_i$ and $t_i$
  - Finite element modeling
  - Complicated ray tracing
  - Experimental verification
Heat Shield for Space Applications

Residual Absorption

\[ u = \text{temperature} \]

\[ Q \]

Heat Sink

Baffles

A: Night side (Blackbody at 90 K)
B: Sunrise point
C: Polar point (Dawn of day)
D: Morning point
E: Zenith point (Blackbody at 688 K + albedo)
Heating by Absorption in the Coating

Boundary conditions

\[ u(r,0) = u_0 \]

\[ D \cdot \frac{\partial u}{\partial r}(R,t) + u(R,t) = u_0 \]

Equation with source term

\[ \frac{\partial u}{\partial t} = \kappa \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{q}{\lambda h} H(t-t_0) \right] \]

Solution

\[ \tilde{u}(\tilde{r},\tilde{t}) = \begin{cases} \frac{\tilde{q}}{4} (1 + 2 \tilde{D} - \tilde{r}^2) - \sum_{j=1}^{\infty} \frac{2\tilde{q} e^{-k_j^2 \tilde{r}}}{(\tilde{D}^2 k_j^2 + 1) k_j^3 J_1(k_j)} \times J_0(k_j \tilde{r}), & \tilde{t} < \tilde{t}_0 \\ \sum_{j=1}^{\infty} \frac{2\tilde{q}}{(\tilde{D}^2 k_j^2 + 1) k_j^3 J_1(k_j)} \left( e^{-k_j^2 (\tilde{t} - \tilde{t}_0)} - e^{-k_j^2 \tilde{t}} \right) J_0(k_j \tilde{r}), & \tilde{t} > \tilde{t}_0 \end{cases} \]

\[ t_0 = T, \quad D = R/10 \]

\[ \tilde{u}(\tilde{r},\tilde{t}) = \frac{u(r,t) - u_0}{U} \]

\[ U = \frac{qR}{\lambda}, \quad \tilde{q} = \frac{q}{R}, \quad \tilde{D} = \frac{D}{R} \]
Concluding Remarks

• The procedure is extended to other applications
  • Stationary problem for spherical lens

• The mathematical theory is submitted to the journal
  • „American Mathematical Monthly“, B. Aebischer, May 2004

• The Presentation and the Matlab programs can be downloaded
  • http://www.leica-geosystems.com/ctc/heat_conduction